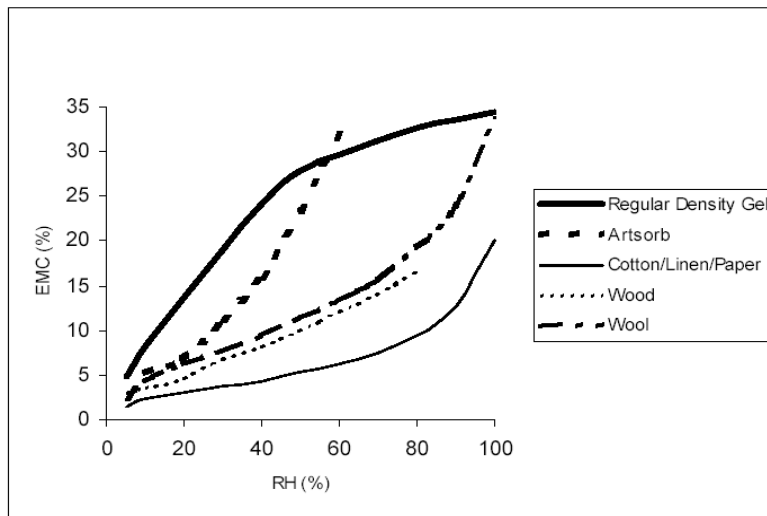


Silica Gel

Silica Gel is a colloidal form of silica (SiO_2) and usually resembles coarse white sand. It is used as a desiccant in food products, electronics and other packaging.

In order to understand how silica gel functions, it is critical to understand the concept of Equilibrium Moisture Content (EMC). Many materials contain moisture. The quantity of moisture in hygroscopic materials (ones that absorb water) depends on the temperature and relative humidity (RH) of the surrounding air. If the temperature or RH changes, the moisture content within the object will change so that it will come into equilibrium with the new condition of the surrounding air. Moisture content is the weight of water in an object expressed as a percentage of its dry weight. The EMC is the moisture content of an object in equilibrium with a specified RH. For example, if a piece of paper weighing 100 grams at 0% RH increases to 105 grams at 50% RH, it now has 5 grams of moisture compared to its dry weight, resulting in a 5% EMC at 50% RH: $(105 \text{ g at } 50\% \text{ RH} - 100 \text{ g at } 0\% \text{ RH}) / 100 \text{ g (the dry weight)} = 0.05 = 5\% \text{ EMC}$.

To understand the moisture uptake characteristics of hygroscopic materials, a series of EMC values for the full range of relative humidity conditions at a fixed temperature can be plotted. This is known as an EMC/RH isotherm. Silica gel goes from roughly 5% to 35% EMC in a non linear fashion:



So really, silica gel is just better at absorbing water than other materials (www.apsnyc.com/pdf/silica_gel_SW_2003.pdf).

Scientific silica gel is sold by Merck Chemicals Ltd for example. Merck's literature indicates a bulk density in g/ml of approximately 0.45 and a particle size range of [μm] 40 – 63 (Cat. No. 1.9385) or 63 – 200 (Cat. No. 1.7734). The density of natural quartz (which is also SiO_2) is 2.635-2.660, so the density specification includes the air spaces within and around the silica particles.

Calculate the percentage composition (%silica & %air) of silica gel at 15°C and 760mm pressure, if the density of silica gel is 0.45g/ml and the density of solid silica is 2.64 and the density of air is given by:

$$d = (P/760) * 0.001293 / (1 + 0.00367t)$$

where P is the pressure in mm of mercury, and t is the temperature in °C.

The density of air is:

$$d = (760/760) * 0.001293 / (1 + 0.00367 \times 15) \\ = 0.001226 \text{ g/ml}$$

Let q be the fraction of silica ($0 \leq q \leq 1$), then:

$$0.45 = 2.64q + 0.001226(1 - q) \\ 0.45 = (2.64 - 0.001226)q + 0.001226 \\ q = (0.45 - 0.001226) / (2.64 - 0.001226) \\ q = 0.17$$

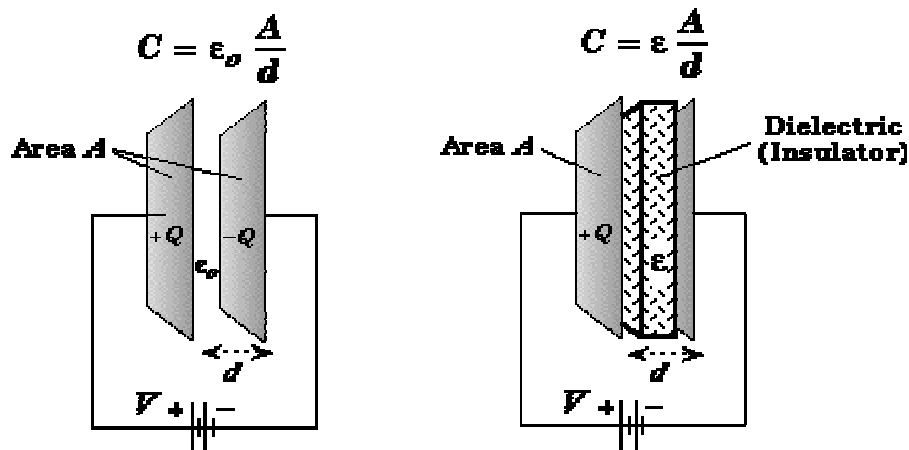
So the composition is 17% silica and 83% air (they add up to 100%).

Parallel Plate Capacitance

The capacitance of two parallel plates of area A separated by a distance d is a linear function of the electrical permittivity of the material between them:

$$C = \epsilon A / d$$

The permittivity is usually expressed as a product of the permittivity of free space ϵ_0 ($8.854 \times 10^{-12} \text{ C}^2 / \text{N.m}^2$) and a dimensionless constant called the dielectric coefficient or dielectric constant which is different for every material.



The dielectric coefficient of bulk silica is 4.27 (parallel to the optic axis) and 4.34 (perpendicular to the optic axis). The dielectric coefficient of air is 1.000590 at 760mm pressure and the dielectric constant of water varies from about 87.8 at 0° to about 55.6 at 100°.

Calculate the permittivity of a uniform mixture of 83% air and 17% silica, given that the dielectric coefficient of silica is 4.3 and that of air is 1 (assume the mixture has a dielectric coefficient that is the weighted average of the two composite values).

$$\epsilon = \frac{1}{2}(0.17 \times 4.3 + 0.83 \times 1) \epsilon_0$$

$$\epsilon = 0.78 \times 8.854 \times 10^{-12}$$

$$\epsilon = 6.9 \times 10^{-12}$$

Calculate the capacitance of two 10cm x 10cm square parallel plates separated by 1 mm of the uniform mixture (above).

$$C = 6.9 \times 10^{-12} \times .01 \times .01 / .001$$

$$= 6.9 \times 10^{-13}$$

$$= 0.69 \text{ pF}$$

Calculate the permittivity of the same mixture with a 35% EMC at 15°C (assume the mixture has a dielectric coefficient that is the weighted average of the three composite values).

Let p be the unknown water mass, and q the unknown silica mass, then assuming air is relatively weightless:

$$0.35 = (p - q) / q$$

$$0.35q = p - q$$

$$p = 1.35q$$

For 17% silica, the water content would be $1.35 \times 0.17 = 23\%$.

Now, the dielectric constant of water can be approximated using a linear interpolation:

$$\epsilon_w = 87.8 + (55.6 - 87.8) \times 15 / 100$$

$$= 83$$

Then the permittivity is:

$$\epsilon = \frac{1}{3}(0.17 \times 4.3 + 0.23 \times 83 + 0.60 \times 1.000590) \epsilon_0$$

$$\epsilon = 20.4 \times 8.854 \times 10^{-12}$$

$$\epsilon = 1.8 \times 10^{-10}$$

Calculate the capacitance of the same parallel plate capacitor with the 35% EMC mixture.

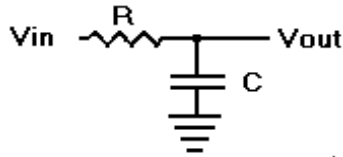
$$C = 1.8 \times 10^{-10} \times .01 \times .01 / .001$$

$$= 1.8 \times 10^{-11}$$

$$= 18 \text{ pF}$$

RC Circuits

A simple RC (resistance capacitance) circuit is shown below:

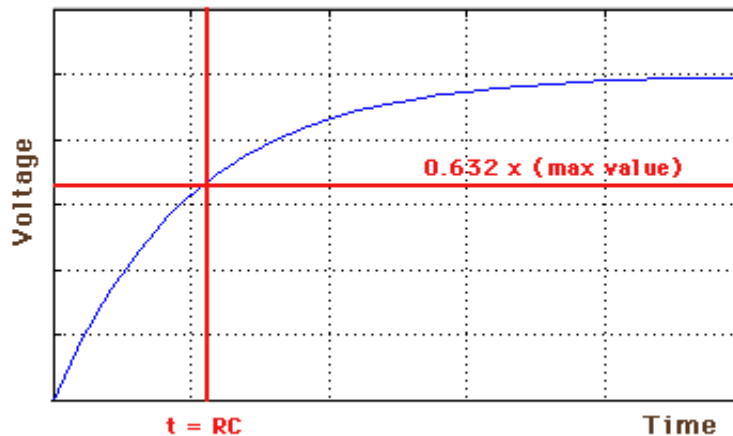


The output voltage (assuming no current is drawn by the load connected to V_{out}) is an exponential function of time after the input voltage is applied:

$$V_{out} = V_{in} * (1 - e^{-(t/RC)})$$

where e is the base of the natural logarithms (approximately 2.71828), t is the time and R and C are the resistance and capacitance respectively.

The product RC is called the time constant and can be thought of as the time required to charge the capacitor to 63% of the applied voltage:



Calculate the time constants for a one megaohm resistor and both a 1pF and 18pF capacitor:

The time constant for the 1pF capacitor is

$$T = R * C$$

$$T = 1 \times 10^{-6} \times 1 \times 10^{-12}$$

$$T = 1 \mu s$$

The time constant for the 18pF capacitor is

$$T = 1 \times 10^{-6} \times 18 \times 10^{-12}$$

$$T = 18 \mu s$$

As a rough guide to the duration of these time constants, a current generation microprocessor in a desktop PC that is clocked at 1.4Ghz can execute roughly 1400 instructions while waiting for 1 μ s to pass. A much more modest microcontroller clocked

at 7MHz could only execute 7 instructions in the same time.

Calculate the output voltage of the above circuit after 3 μ s if the input voltage is 5 Volts and the resistor is 1MegaOhm, for two different values of capacitance 0.69pF and 18pF.

$$\begin{aligned}V_{\text{out}} &= 5.0 * (1 - e^{-(3 \times 10^{-6} / 1 \times 10^6 \times 0.69 \times 10^{-12})}) \\ &= 5.0 * 0.987 \\ &= 4.94\end{aligned}$$

$$\begin{aligned}V_{\text{out}} &= 5.0 * (1 - e^{-(3 \times 10^{-6} / 1 \times 10^6 \times 18 \times 10^{-12})}) \\ &= 5.0 * .154 \\ &= 0.768\end{aligned}$$

Synergy

Sketch an instrument that would produce a voltage proportional to humidity and outline a strategy for measurement based on a microprocessor.